Hypothesis Testing

The idea of hypothesis testing is:

- Ask a question with two possible answers
- Design a test, or calculation of data
- Base the decision (answer) on the test

Example: In 2010, 24% of children were dressed as Justin Bieber for Halloween. We want to test whether or not this proportion increased in 2011.

Constructing a Hypothesis Test

- Define your Null and Alternative Hypotheses
- H_0 (pronounced “H naught”) is the **null hypothesis**. This is typically the default assumption – assuming no change, or that a new drug is no better than a placebo.
- H_A is the **alternative hypothesis**. This is typically your hunch, that there has been a change, or that the drug works better than the placebo, or a claim that you are trying to debunk.

Example: Justin Bieber Costumes

H_0: In 2011, 24% of Halloween costumes were Bieber costumes. (**p=.24)**

H_A: In 2011, the proportion of Bieber costumes was greater than 24%; **(p>.24)**

Alternative Hypothesis

- The Alternative Hypothesis is chosen to match a claim that is being tested, or something you hope is true.
- Say we are testing for a proportion p. Null Hypothesis is $p=p_0$.
- If the Alternative Hypothesis is $p>p_0$, or if it is $p<p_0$, these are examples of a **one-sided test**.
- If the alternative is $p \neq p_0$, this is called a **two-sided test**.

Calculate a Test Statistic

- For a hypothesis test about population proportion, sample proportion is a good test statistic (if the conditions of the CLT are met, we can use the normal distribution)

Example: We randomly poll 1000 children who dressed up for Halloween in 2011. 254 of them dressed up as Justin Bieber, so our sample proportion is .254

P-Value

- For a hypothesis test of a proportion, we use a P-Value. This is the conditional probability of the tails assuming H_0 is true.
- The smaller the P-value, the more strong the evidence in favor of our alternative Hypothesis.
- If the P-Value is less than or equal to a certain predefined threshold (the **significance level**), we will reject the null Hypothesis.

Calculating P-Values (1-sided tests)

- $\hat{p}$ is normal with $\mu=p_0$, $\sigma=\sqrt{(p_0(1-p_0)/n)}$
- Calculate x/n, the proportion from our observed sample.
- Case1: H_0: $p=p_0$ vs. H_A: $p>p_0$

  P-Value= $P[\hat{p}>x/n \mid p=p_0]$ =normalcdf(x/n,1,μ,σ)
- Case2: H_0: $p=p_0$ vs. H_A: $p<p_0$

  P-Value= $P[\hat{p}<x/n \mid p=p_0]$ =normalcdf(0,x/n,μ,σ)

Example: Calculate P-Value

We are using a 5% significance level.

- If we assume H_0 true, then $\hat{p}$, our sample proportion, is normal with μ=.24 and σ=√(.24*.76/1000)=.0135
- $P[\hat{p}>.254|H_0]$ =`normalcdf(.254,1,.24,.0134)` =.1481
- This is not less than our .05 significance level, so we do not reject the null hypothesis

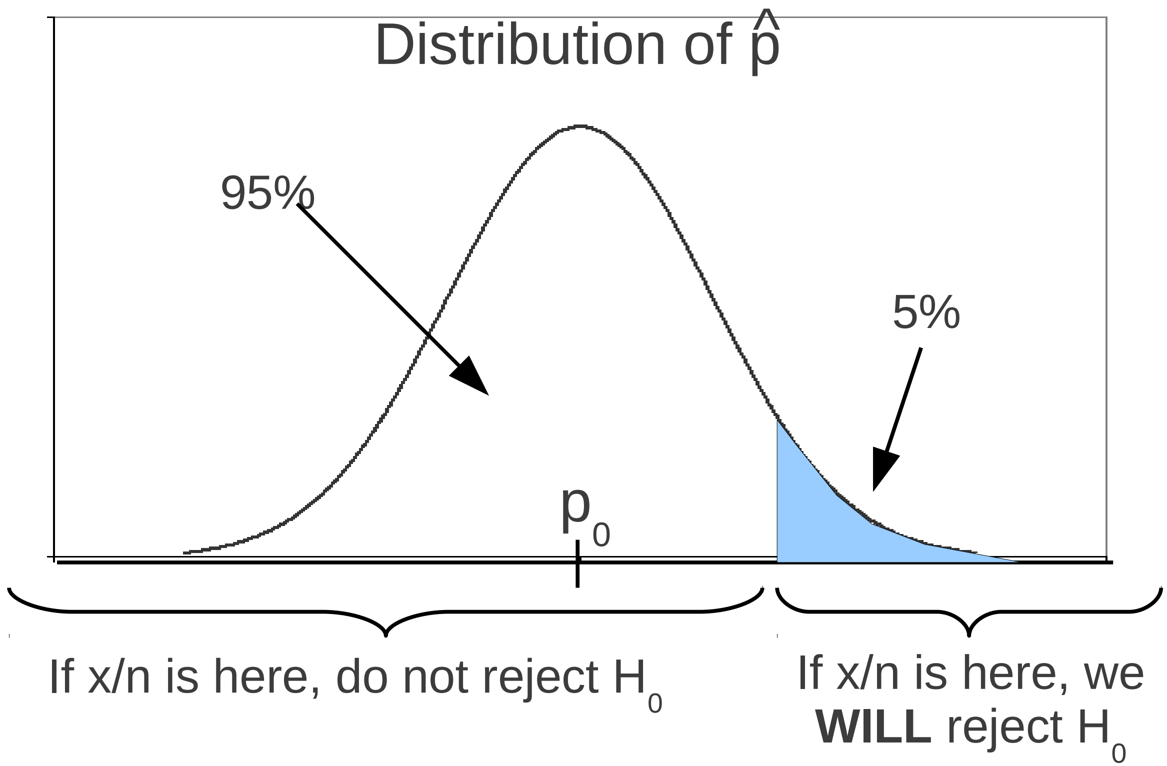
The evidence is not strong enough to support the claim that Justin Bieber costumes were more popular in 2011 than in 2010.

Visual Hypothesis Test

H_0: $p=p_0$ vs. H_A: $p>p_0$ 5% significance level

By CLT, $\hat{p}$ is normal with $\mu=p_0$, $\sigma=\sqrt{(p_0q_0/n)}$
(Assuming H_0 is true)

Calculate x/n, our observed sample proportion

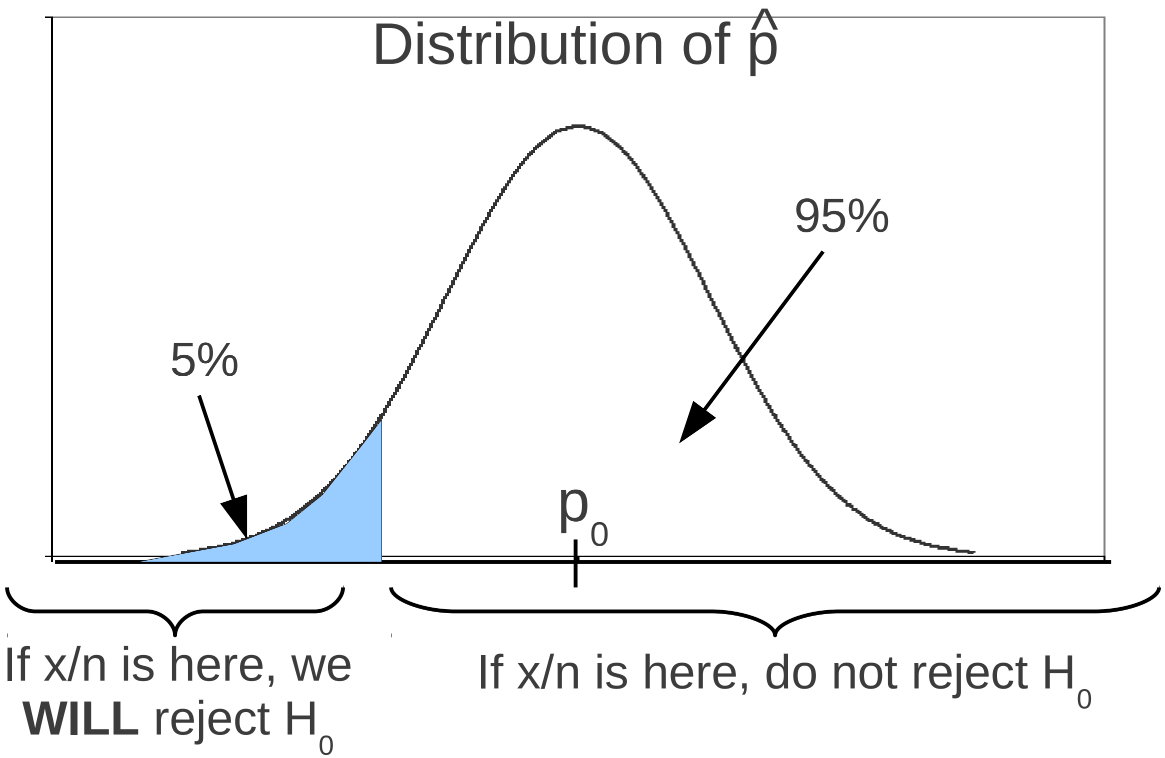


Visual Hypothesis Test

H_0: $p=p_0$ vs. H_A: $p<p_0$ 5% significance level

By CLT, $\hat{p}$ is normal with $\mu=p_0$, $\sigma=\sqrt{(p_0 q_0/n)}$
(Assuming H_0 is true)

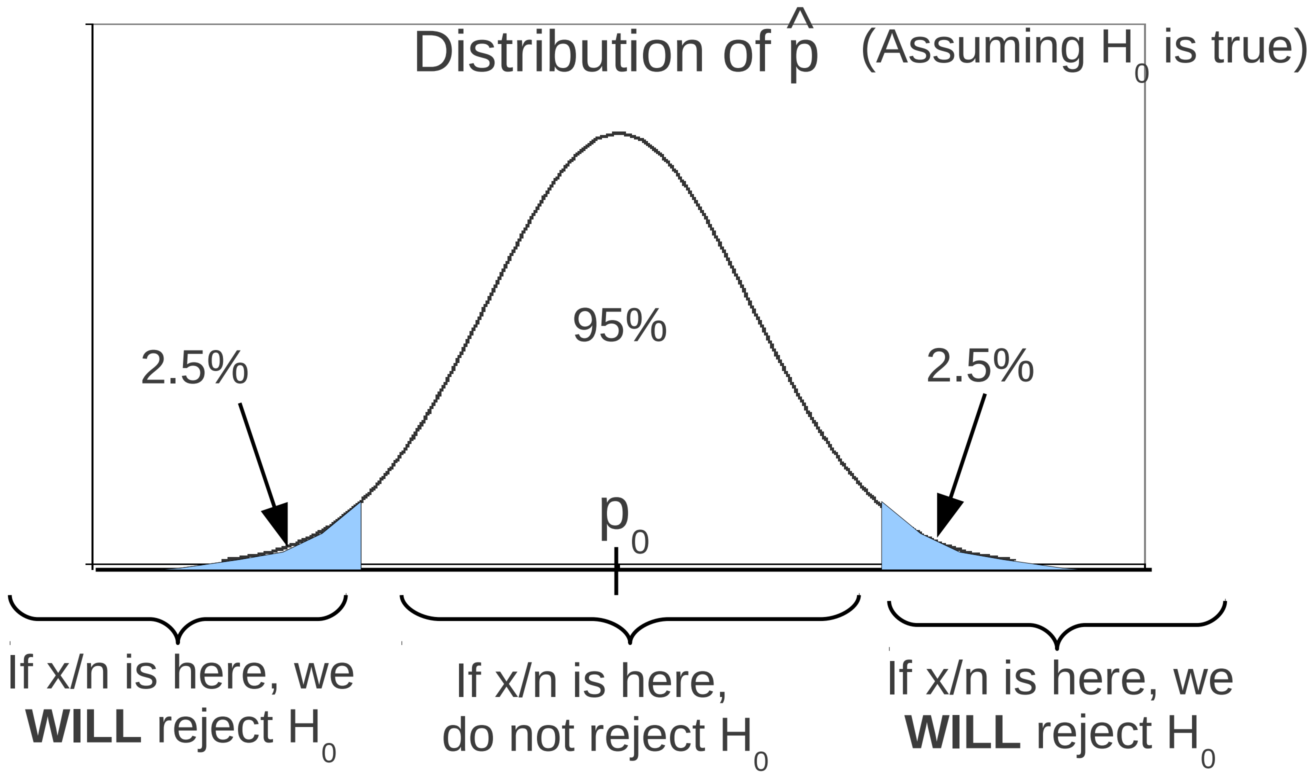
Calculate x/n, our observed sample proportion



2-Sided Test

- A Two tailed test looks like this:

H_0: $p=p_0$ vs. H_A:$p\neq p_0$, 5% significance level



Calculating P-Values (2-sided tests)

- $\hat{p}$ is normal with $\mu=p_0$, $\sigma=\sqrt{(p_0 q_0/n)}$
- Calculate x/n, the proportion from our observed sample.

H_0: $p=p_0$ vs. H_A: $p \neq p_0$

use standardized z value $z=\dfrac{\frac{x}{n}-\mu}{\sigma}=\dfrac{\frac{x}{n}-p_0}{\sqrt{\frac{p_0 q_0}{n}}}$

- P-Value= $2P[Z>|z| \mid \hat{p}=p_0]$ =2*normalcdf(|z|,6)
- Notice we use |z| - the absolute value of z

Example- 2-sided test

In 1996, 25% of students who had perfect attendance one month would also have perfect attendance the following month. In 2000, the school wants to see if the proportion has changed. The proportion of a random sample of 6543 students is 23.4%. With a 5% significance level, should the school conclude that there has been a change?

- H_0: proportion is still 25%, H_A: proportion is not 25%
- Under the null hypothesis, $\hat{p}$ is normal with mean 0.25 and s.d √(.25*.75/6453)=.00535
- Standardized proportion is z=(.234-.25)/.00535 =-2.9907
- P value is 2*P[Z>2.9907] = 2*normalcdf(2.9907,6)=.0028
- **This is less than .05, so we reject the null hypothesis; there has been a change in attendance.**

Test using Z-Statistic

You can calculate the P-Value using a z-score:

Assume null hypothesis: $\boldsymbol{\mu = p_0, \sigma = \sqrt{(p_0 q_0 / n)}}$

X/n is our sample proportion. $\boldsymbol{z = (x - \mu)/\sigma}$

The P-value depends on the form of H_A:

- $H_A: p > p_0 \rightarrow$ P-Value = P(Z>z)=normalcdf(z,6)
- $H_A: p < p_0 \rightarrow$ P-Value = P(Z<z)=normalcdf(-6,z)
- $H_A: p \neq p_0 \rightarrow$ P-Value = 2P(Z>|z|)=2*normalcdf(|z|,6)

Remember, for 2-tailed test use **|z|** (absolute value)

One-Sided Test using Z-statistic

A Magazine wants to launch an online version, but only if more than 20% of its subscribers would subscribe to it. A random survey of 400 subscribers indicated that 90 would be interested.

- H_0: p=20%, H_A: p>20%, assume 5% significance level
- Sample proportion is 90/400=.225
- Under the null hypothesis, $\hat{p}$ is normal with mean 0.20 and s.d $\sqrt{(.20*.80/400)}$=.02
- Standardized proportion is z=(.225-.20)/.02 =1.25
- P value is P[Z>1.25] = normalcdf(1.25,6)=.10565
- **This is higher than our .05 significance level; evidence is not strong enough to reject the null hypothesis. The magazine should not launch the online version.**